Chapter 20. Area and Perimeter of Plane Figures

Exercise 20(A)

Solution 1:

Since the sides of the triangle are 18cm,24cm and 30cm respectively.

$$s = \frac{18 + 24 + 30}{2}$$
$$= 36$$

Hence area of the triangle is

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{36(36-18)(36-24)(36-30)}$$
$$= \sqrt{36 \times 18 \times 12 \times 6}$$
$$= \sqrt{46656}$$
$$= 216 \text{sqcm}$$

Again

$$A = \frac{1}{2} \text{base} \times \text{altitude}$$

Hence

$$216 = \frac{1}{2} \times 30 \times h$$
$$h = 14.4cm$$

Solution 2:

Let the sides of the triangle are

a=3x

b=4x

c=5x

Given that the perimeter is 144 cm.

hence

$$3x + 4x + 5x = 144$$
$$\Rightarrow 12x = 144$$
$$\Rightarrow x = \frac{144}{12}$$
$$\Rightarrow x = 12$$
$$s = \frac{a+b+c}{2} = \frac{12x}{2} = 6x = 72$$

The sides are a=36 cm, b=48 cm and c=60 cm

Area of the triangle is

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{72(72-36)(72-48)(72-60)}$$
$$= \sqrt{72 \times 36 \times 24 \times 12}$$
$$= \sqrt{746496}$$
$$= 864\ cm^2$$

Solution 3:

(i)

Area of the triangle is given by

$$A = \frac{1}{2} \times AB \times AC$$
$$= \frac{1}{2} \times 4 \times 4$$
$$= 8 \text{ sq.cm}$$

(ii)

Again area of the triangle

$$A = \frac{1}{2} \times BC \times h$$
$$8 = \frac{1}{2} \times \left(\sqrt{4^2 + 4^2}\right) \times h$$
$$h = 2.83 cm$$

Solution 4:

Area of an equilateral triangle is given by

$$\frac{\sqrt{3}}{4} \times (side)^2 = A$$
$$\frac{\sqrt{3}}{4} \times (side)^2 = 36\sqrt{3}$$
$$(side)^2 = 144$$
$$side = 12\ cm$$

Hence

$$\text{perimeter} = 3 \times (\text{its side})$$
$$= 3 \times 12$$
$$= 36\ cm$$

Solution 5:

Since the perimeter of the isosceles triangle is 36cm and base is 16cm.

hence the length of each of equal sides are $\frac{36-16}{2} = 10cm$

Here

It is given that

a = equa lsides = $10cm$

b = base = $16cm$

Let 'h' be the altitude of the isosceles triangle.

Since the altitude from the vertex bisects the base perpendicularly, we can apply Pythagoras Theorem.

Thus we have,

$$h = \sqrt{a^2 - \left(\frac{b}{2}\right)^2} = \frac{1}{2}\sqrt{4a^2 - b^2}$$

We know that

$$Area\ of\ the\ triangle = \frac{1}{2} \times base \times altitude$$

$$\text{Area of the triangle} = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

$$= \frac{1}{4} \times 16 \times \sqrt{400 - 256}$$

$$= 48\text{sq.cm}$$

Solution 6:

It is given that

$Area = 192\ sq.cm$

$base = 24\ cm$

Knowing the length of equal side, a, and base, b, of an isosceles triangle, the area can be calculated using the formula,

$$A = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

Let 'a' be the length of an equal side.

$$\text{Area} = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

$$192 = \frac{1}{4} \times 24 \times \sqrt{4a^2 - 576}$$

$$192 = 6\sqrt{4a^2 - 576}$$

$$\sqrt{4a^2 - 576} = 32$$

$$4a^2 - 576 = 1024$$

$$4a^2 = 1600$$

$$a = 20cm$$

Hence perimeter= $20 + 20 + 24 = 64cm$

Solution 7:

From ΔABC,

$$AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{16^2 - 8^2}$$
$$= \sqrt{192}$$

Area of ΔABC

$$\Delta ABC = \frac{1}{2} \times 8 \times \sqrt{192}$$
$$= 4\sqrt{192}$$

Area of ΔBCD

$$\Delta BCD = \frac{\sqrt{3}}{4} \times 8^2$$
$$= 16\sqrt{3}$$

Now

$$\Delta ABD = \Delta ABC - \Delta BDC$$
$$= 4\sqrt{192} - 16\sqrt{3}$$
$$= 32\sqrt{3} - 16\sqrt{3}$$
$$= 16\sqrt{3} \text{ sq.cm}$$

Solution 8:

Given , AB = 8 cm, AD = 10 cm, BD = 12 cm, DC = 13 cm and $\angle$ DBC = 90°

$$BC = \sqrt{DC^2 - BD^2}$$
$$= \sqrt{13^2 - 12^2}$$
$$= 5cm$$

Hence perimeter=8+10+13+5=36cm

Area of ΔABD

$$\Delta ABD = \sqrt{15(15-8)(15-10)(15-12)}$$
$$= \sqrt{15 \times 7 \times 5 \times 3}$$
$$= 15\sqrt{7}$$
$$= 39.7$$

Area of ΔDBC

$$\Delta BDC = \frac{1}{2} \times 12 \times 5$$
$$= 30$$

Now

Solution 9:

$$\text{Area of } ABCD = area\ of\ \Delta ABD + area\ of\ \Delta BDC$$
$$= 39.7 + 30$$
$$= 69.7 \text{sq. cm}$$

$$\text{Area of the rectangular field} = \frac{49572}{36.72} = 135000$$

Let the height of the triangle be x

$$135000 = \frac{1}{2} \times x \times 3x$$
$$\Rightarrow x^2 = 90000$$
$$\Rightarrow x = 300$$

Height $= 300$ m

Base $= 900$ m

Solution 10:

(i)

Given that the sides of a triangle are in the ratio 5:3:4.

Also, given that the perimeter of the triangle is 180.

Thus, we have, $5x + 4x + 3x = 180$

$\Rightarrow 12x = 180$

$\Rightarrow x = \frac{180}{12}$

$\Rightarrow x = 15$

Thus, the sides are 5×15, 3×15 and 4×15.

That is the sides are 75 m, 45 m and 60 m.

Since the sides are in the ratio, 5:3:4, it is a Pythagorean triplet.

Therefore, the triangle is a right angled triangle.

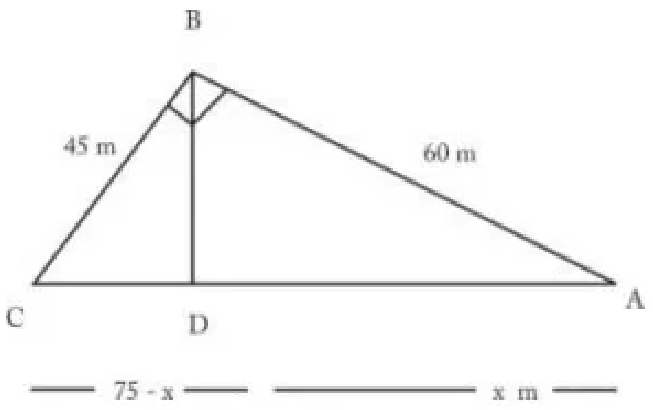
Area of a right angled triangle $= \frac{1}{2} \times \text{base} \times \text{altitude}$

$\Rightarrow = \frac{1}{2} \times 45 \times 60$

$\Rightarrow = 45 \times 30 = 1350\ m^2$

(ii)

Consider the following figure.



In the above figure,
The largest side is AC = 75 m.
The altitude corresponding to AC is BD.
We need to find the value of BD.
Now consider the triangles ΔBCD and ΔBAD.
We have,

$\angle B = \angle B$ [common]

$BD = BD$ [common]

$\angle D = \angle D = 90°$

Thus, by Angle-Side-Angle criterion of congruence, we have ΔBCD ~ ΔABD.
Similar triangles have similar proportionality.
Thus, we have,

$$\frac{\text{CD}}{\text{BD}} = \frac{BD}{AD}$$

$$\Rightarrow BD^2 = AD \times CD \ldots(1)$$

From subpart(i), the sides of the triangle are AC = 75 m, AB = 60 m and BC = 45 m
Let AD = x m ⇒ CD= $(75 - x)$ m
Thus applying Pythgoras Theroem, from right triangle ΔBCD, we have

$$45^2 = (75 - x)^2 + BD^2$$

$$\Rightarrow BD^2 = 45^2 - (75 - x)^2$$

$$\Rightarrow BD^2 = 2025 - (5625 + x^2 - 150x)$$

$$\Rightarrow BD^2 = 2025 - 5625 - x^2 + 150x$$

$$\Rightarrow BD^2 = -3600 - x^2 + 150x \ldots(2)$$

Now applying Pythgoras Theroem, from right triangle ΔABD, we have

$$60^2 = x^2 + BD^2$$

$$\Rightarrow BD^2 = 60^2 - x^2$$

$$\Rightarrow BD^2 = 3600 - x^2 \ldots(3)$$

From equations (2) and (3), we have,

$$-3600 - x^2 + 150x = 3600 - x^2$$

$$\Rightarrow 150x = 3600 + 3600$$

$$\Rightarrow 150x = 7200$$

$\Rightarrow x = \frac{7200}{150}$

$\Rightarrow x = 48$

Thus, AD = 48 and CD = 75 – 48 = 27

Substituting the values AD=48 m

and CD=27 m in equation (1), we have

$BD^2 = 48 \times 27$

$\Rightarrow BD^2 = 1296$

$\Rightarrow BD = 36$ m

The altitude of the triangle corresponding to its largest side is BD = 36 m

(iii)

The area of the triangular field from subpart(i) is 1350 m^2

The cost of levelling the field is Rs.10 per square metre.

Thus, the total cost of levelling the field = 1350×10 = Rs.13,500

Solution 11:

Let the height of the triangle be x cm.

Equal sides are (x+4) cm.

According to Pythagoras theorem,

$$(x+4)^2 = x^2 + 12^2$$

$$8x = 128$$

$$x = 16cm$$

Hence perimeter= $20 + 20 + 24 = 64cm$

Area of the isosceles triangle is given by

Here a=20cm

b=24cm

hence

$$Area = \frac{1}{4} \times b \times \sqrt{4a^2 - b^2}$$

$$= \frac{1}{4} \times 24 \times \sqrt{1024}$$

$$= 192 sq.cm$$

Solution 12:

Each side of the triangle is $\frac{60}{3} = 20cm$

Hence the area of the equilateral triangle is given by

$$A = \frac{\sqrt{3}}{4} \times 20^2$$
$$= 100\sqrt{3}$$
$$= 173.2 sq.cm$$

The height h of the triangle is given by

$$\frac{1}{2} \times 20 \times h = 173.2$$
$$h = 17.32cm$$

Solution 13:

The area of the triangle is given as 150sq.cm

$$\frac{1}{2} \times x \times (x+5) = 150$$
$$x^2 + 5x - 300 = 0$$
$$(x+20)(x-15) = 0$$
$$x = 15$$

Hence AB=15cm,AC=20cm and

$$BC = \sqrt{15^2 + 20^2}$$
$$= 25cm$$

Solution 14:

Let the two sides be x cm and (x-3) cm.

Now

$$\frac{1}{2}\times x\times(x-3)=54$$

$$x^2-3x-108=0$$

$$(x-12)(x+9)=0$$

$$x=12cm$$

Hence the sides are 12cm, 9cm and $\sqrt{12^2+9^2}=15cm$

The required perimeter is 12+9+15=36cm.

Solution 15:

$$\text{Area of }\Delta ABC=\frac{1}{4}\times 36\times\sqrt{4\times 30^2-36^2}$$

$$=\frac{1}{4}\times 36\times\sqrt{2304}$$

$$=\frac{1}{4}\times 36\times 48$$

$$=432$$

Since AB=AC and $\angle BOC=90^\circ$

$$\angle BOD=\angle COD=45^\circ$$

$$\text{hence }\angle OBD=45^\circ\text{ and }OD=BD=18cm$$

Now

$$\text{Area of }\Delta BOC=\frac{1}{2}\times 36\times 18$$

$$=324$$

$$\text{Area of }ABOC=\text{Area of }\Delta ABC-\text{Area of }\Delta BOC$$

$$=432-324$$

$$=108\text{sq.cm}$$

Exercise 20(B)

Solution 1:

$$\text{Area} = \frac{1}{2} \times \text{one diagonal} \times \text{sum of the lengths of the}$$

perpendiculars drawn from it on the remaining

two vertices.

$$= \frac{1}{2} \times 30 \times (11 + 19)$$

$$= 450 \text{sq.cm}$$

Solution 2:

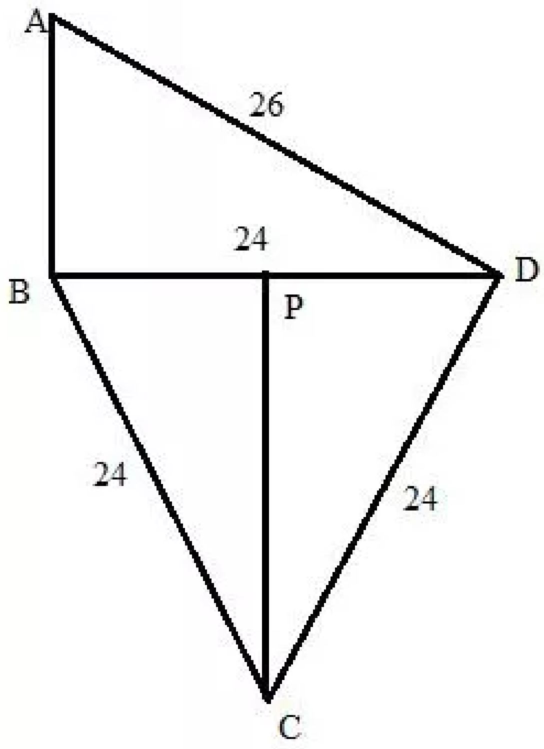
$$\text{Area of the quadriletaral} = \frac{1}{2} \times \text{the product of the diagonals.}$$

$$= \frac{1}{2} \times 16 \times 13$$

$$= 104 cm^2$$

Solution 3:

Consider the figure:



From the right triangle *ABD* we have

$$
\begin{aligned}
AB &= \sqrt{26^2 - 24^2} \\
&= 2\sqrt{13^2 - 12^2} \\
&= 2(5) \\
&= 10
\end{aligned}
$$

The area of right triangle *ABD* will be:

$$
\begin{aligned}
\Delta ABD &= \frac{1}{2}(AB)(BD) \\
&= \frac{1}{2}(10)(24) \\
&= 120
\end{aligned}
$$

Again from the equilateral triangle *BCD* we have $CP \perp BD$

$$\begin{aligned} PC &= \sqrt{24^2 - 12^2} \\ &= 12\sqrt{2^2 - 1^1} \\ &= 12\sqrt{3} \end{aligned}$$

Therefore the area of the triangle *BCD* will be:

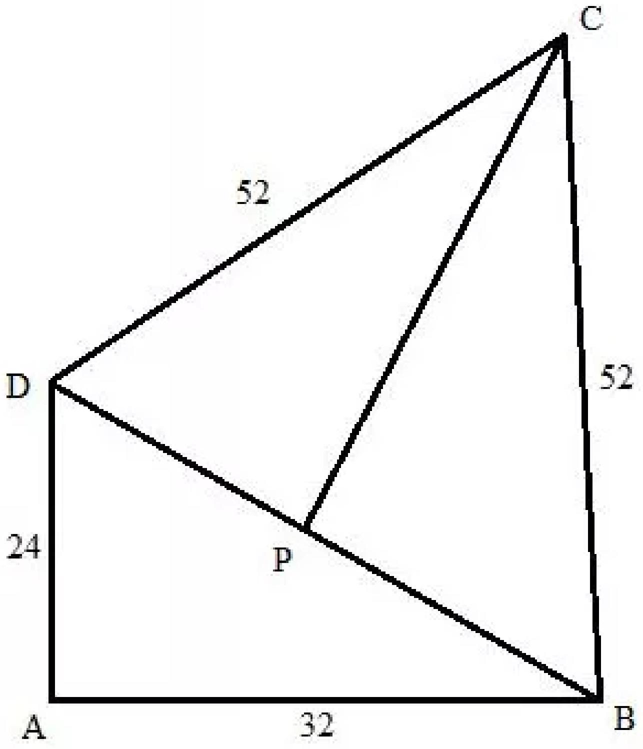
$$\begin{aligned} \Delta BCD &= \frac{1}{2}(BD)(PC) \\ &= \frac{1}{2}(24)\left(12\sqrt{3}\right) \\ &= 144\sqrt{3} \end{aligned}$$

Hence the area of the quadrilateral will be:

$$\begin{aligned} \Delta ABD + \Delta BCD &= 120 + 144\sqrt{3} \\ &= 369.41 \text{ cm}^2 \end{aligned}$$

Solution 4:

The figure can be drawn as follows:



Here *ABD* is a right triangle. So the area will be:

$$\Delta ABD = \frac{1}{2}(24)(32)$$
$$= 384$$

Again

$$BD = \sqrt{24^2 + 32^2}$$
$$= 8\sqrt{3^2 + 4^2}$$
$$= 8(5)$$
$$= 40$$

Now BCD is an isosceles triangle and BP is perpendicular to BD, therefore

$$DP = \frac{1}{2} BD$$
$$= \frac{1}{2}(40)$$
$$= 20$$

From the right triangle DPC we have

$$PC = \sqrt{52^2 - 20^2}$$
$$= 4\sqrt{13^2 - 5^2}$$
$$= 4(12)$$
$$= 48$$

So

$$\Delta DPC = \frac{1}{2}(40)(48)$$
$$= 960$$

Hence the area of the quadrilateral will be:

$$\Delta ABD + \Delta DPC = 960 + 384$$
$$= 1344 \text{ cm}^2$$

Solution 5:

Let the width be x and length 2x km.

Hence

$$2(x + 2x) = \frac{3}{5}$$
$$x = \frac{1}{10} km$$
$$= 100m$$

Hence the width is 100m and length is 200m

The required area is given by

$$A = \text{length} \times \text{width}$$
$$= 100 \times 200$$
$$= 20{,}000 \text{sq.m}$$

Solution 6:

Length of the laid with grass=85-5-5=75m

Width of the laid with grass=60-5-5=50m

Hence area of the laid with grass is given by

$$A = 75 \times 50$$

$$= 3750 \text{sq.m}$$

Solution 7:

Area of the rectangle is given by

$$A = l \times b$$

$$= 6 \times 4$$

$$= 24 \text{sq.cm}$$

Let h be the height of the triangle ,then

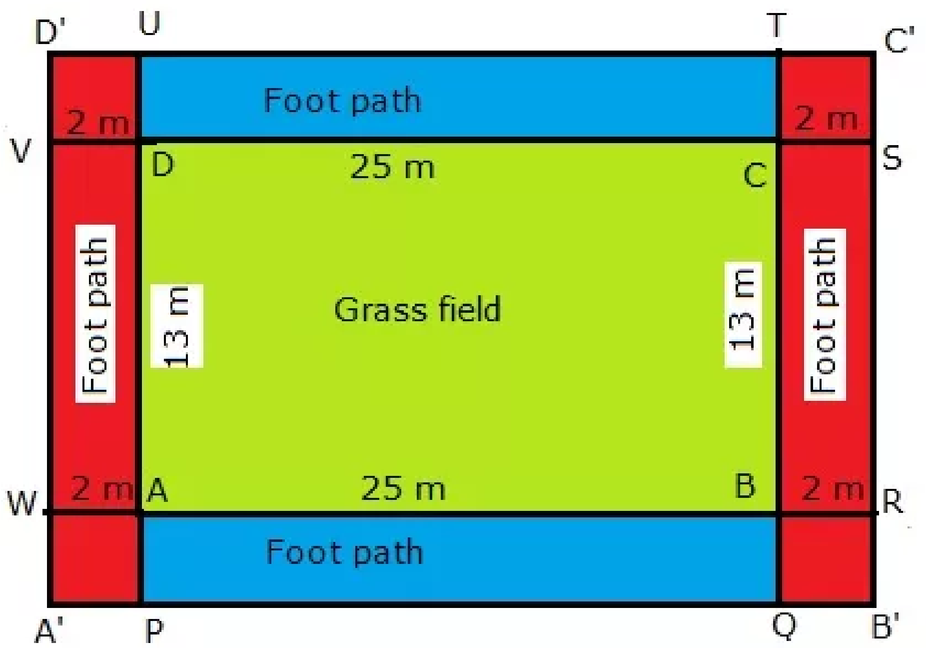
$$\frac{1}{2} \times base \times h = 3A$$

$$\frac{1}{2} \times 6 \times h = 3 \times 24$$

$$h = 24cm$$

Solution 8:

Consider the following figure.



Thus the required area = area shaded in blue + area shaded in red

= Area ABPQ + Area TUDC + Area A'PUD' + Area QB'C'T

= 2Area ABPQ + 2Area QB'C'T

=2(Area ABPQ +Area QB'C'T)

Area of the footpath is given by

$$A = 2 \times (25 + 25 + 17 + 17)$$
$$= 168 \ sq.m$$
$$= 1680000 \ sq.cm$$

$$\text{Hence number of tiles required} = \frac{1680000}{400} = 4200$$

Solution 9:

Perimeter of the garden

$$s = \frac{300}{0.75}$$
$$= 400\text{sq.m}$$

Again, length of the garden is given to be 120 m. hence breadth of the garden b is given by

$$2(l+b) = S$$
$$2(120+b) = 400$$
$$b = 80m$$

Hence area of the field

$$A = 120 \times 80$$
$$= 9600\text{sq.m}$$

Solution 10:

Length of the rectangle=x

Width of the rectangle= $\frac{4}{7}x$

Hence its perimeter is given by

$$2\left(x+\frac{4}{7}x\right) = y$$
$$2\left(\frac{11x}{7}\right) = y$$
$$\frac{22x}{7} = y$$

Again it is given that the perimeter is 4400cm.

Hence

$$\frac{22x}{7} = 4400$$
$$x = 1400$$

Length of the rectangle=1400 cm = 14 m

Solution 11:

(i)

Breadth of the verandah=x

Length of the verandah=x+3

According to the question

$$2(x+(x+3)) = x(x+3)$$
$$4x+6 = x^2 + 3x$$
$$x^2 - x - 6 = 0$$

(ii)

From the above equation

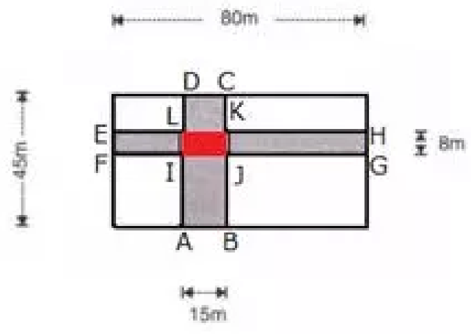
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3$$

Hence breadth=3m

Length =3+3=6m

Solution 12:

Consider the following figure.



Thus, the area of the shaded portion

=Area(ABCD) + Area(EFGH) - Area(IJKL)...(1)

Dimensions of ABCD:45 m×15 m

Thus, the area of ABCD = 45×15 = 675 m^2

Dimensions of EFGH:80 m × 8 m

Thus, the area of EFGH = 80 × 8 = 640 m^2

Dimensions of IJKL:15 m × 8 m

Thus, the area of IJKL = 80 × 8 = 120 m^2

Therefore, from equation (1),

the area of the shaded

portion = 675 + 640 − 120 = 1195 m^2

Solution 13:

First we have to calculate the area of the hall.

$$Area = 45 \times 32$$
$$= 1440 m^2$$

$$Cost = \frac{40}{1.20} \times 1440$$
$$= 48,000$$

We need to find the cost of carpeting of 80 cm = 0.8 m wide carpet, if the rate of carpeting is Rs. 25. Per metre.
Then

$$Cost = \frac{25}{0.8} \times 1440$$
$$= Rs.45,000$$

Solution 14:

Let a be the length of each side of the square.

Hence

$$2a^2 = (diagonal)^2$$
$$a^2 = \frac{15^2}{2}$$
$$a^2 = 112.5$$
$$a = 10.60$$

Hence

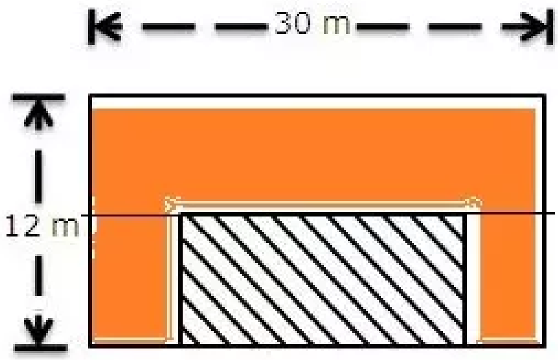
$$\text{Area} = a^2$$
$$= 112.5 \text{sq.m}$$

And

$$\text{Perimeter} = 4a = 42.43 m$$

Solution 15:

Consider the following figure.



(i)

The length of the lawn = 30 - 2 - 2 = 26 m

The breadth of the lawn = 12 - 2 = 10 m

(ii)

The orange shaded area in the figure is the required area.

Area of the flower bed is calculated as follows:

$$A = 10 \times 2 + 10 \times 2 + 30 \times 2$$

$$= 20 + 20 + 60$$

$$= 100 \text{sq.m}$$

Solution 16:

$$\text{Area of the floor} = 15 \times 8$$
$$= 120 \text{sq.m}$$

$$\text{Area of one tiles} = 0.50 \times 0.25$$
$$= 0.125 \text{sq.m}$$

Number of tiles required

$$n = \frac{\text{Area of floor}}{\text{Area of tiles}}$$
$$= \frac{120}{0.125}$$
$$= 960$$

$$\text{Area of carpet uncovered} = 2(1 \times 15 + 1 \times 6)$$
$$= 42 \text{sq.m}$$

Fraction of floor uncovered= $\frac{42}{120} = \frac{7}{20}$

Solution 17:

Since

$$\text{Area} = \text{Base} \times \text{Height}$$
$$\therefore \; 24 \times 12 = 18 \times h$$
$$h = \frac{24 \times 12}{18}$$
$$= 16m$$

Hence the distance between the shorter sides is 16m.

Solution 18:

At first we have to calculate the area of the triangle having sides 10cm,12cm and 16cm. let the area be S.

Now

$$S = \frac{10+12+16}{2}$$

$$= 19\text{cm}$$

$$A = \sqrt{19 \times (19-10) \times (19-12) \times (19-16)}$$

$$= \sqrt{19 \times 9 \times 7 \times 3}$$

$$= 59.9\text{sq.cm}$$

$$\text{Area of parallelogram} = 2A$$

$$= 2 \times 59.9$$

$$= 119.8\text{sq.cm}$$

Again

Area=base x height

Here base=10cm

Hence

$$\text{height} = \frac{\text{Area}}{\text{base}}$$

$$= \frac{119.8}{10}$$

$$= 11.98\text{cm}$$

Solution 19:

(i)

We know that

$$\text{Area of Rhombus}=\frac{1}{2}\times AC\times BD$$

Here A=216sq.cm

AC=24cm

BD=?

Now

$$A=\frac{1}{2}\times AC\times BD$$

$$216=\frac{1}{2}\times 24\times BD$$

$$BD=18\text{cm}$$

(ii)

Let a be the length of each side of the rhombus.

$$a^2=\left(\frac{AC}{2}\right)^2+\left(\frac{BD}{2}\right)^2$$

$$a^2=12^2+9^2$$

$$a^2=225$$

$$a=15\text{cm}$$

(iii)

Perimeter of the rhombus=4a=60cm.

Solution 20:

Let a be the length of each side of the rhombus.

$4a = \text{perimeter}$

$4a = 52$

$a = 13\text{cm}$

(i)

It is given that,

AC=24cm

We have to find BD.

Now

$$a^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$$13^2 = 12^2 + \left(\frac{BD}{2}\right)^2$$

$$\left(\frac{BD}{2}\right)^2 = 5^2$$

$$BD = 10\text{cm}$$

Hence the other diagonal is 10cm.

(ii)

$$\text{Area of the rombus} = \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 24 \times 10$$

$$= 120\text{sq.cm}$$

Solution 21:

Let a be the length of each side of the rhombus.

$$4a = \text{perimeter}$$

$$4a = 46$$

$$a = 11.5\text{cm}$$

We know that,

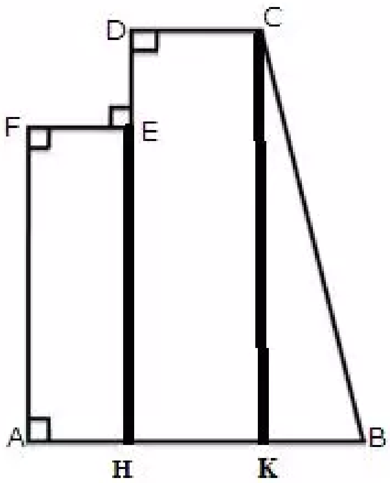
$$\text{Area} = \text{Base}\times\text{Height}$$

$$= 11.5\times 8$$

$$= 92\text{sq.cm}$$

Solution 22:

The diagram is redrawn as follows:



Here

AF=1.2m,EF=0.3m,DC=0.6m,BK=1.8-0.6-0.3=0.9m

Hence

$$\text{Area of } ABCDEF = \text{Area of } AHEF + \text{Area of } HKCD$$

$$+\Delta KBC$$

$$= 1.2\times 0.3 + 2\times 0.6 + \frac{1}{2}\times 2\times 0.9$$

$$= 2.46\text{sq.m}$$

Solution 23:

Here we found two geometrical figure, one is a triangle and other is the trapezium.

Now

$$\text{Area of the triangle} = \frac{1}{2} \times 12 \times 25$$

$$= 150\text{sq.m}$$

$$\text{Area of the trapezium} = \frac{1}{2} \times (25+15) \times \left(\sqrt{26^2 - (25-15)^2}\right)$$

$$= 20 \times 24$$

$$=480\text{sq.m}$$

hence area of the whole figure=150+240=630sq.m

Solution 24:

We can divide the field into three triangles and one trapezium.

Let A,B,C be the three triangular region and D be the trapezoidal region.

Now

$$\text{Area of } A = \frac{1}{2} \times AD \times GE$$

$$= \frac{1}{2} \times (50 + 40 + 15 + 25) \times 60$$

$$= 3900\text{sq.m}$$

$$\text{Area of } B = \frac{1}{2} \times AF \times BF$$

$$= \frac{1}{2} \times 50 \times 50$$

$$= 1250\text{sq.m}$$

$$\text{Area of } B = \frac{1}{2} \times HD \times CH$$

$$= \frac{1}{2} \times 25 \times 25$$

$$= 312.5\text{sq.m}$$

$$\text{Area of } D = \frac{1}{2} \times (BF + CH) \times (FG + GH)$$

$$= \frac{1}{2} \times (50 + 25) \times (40 + 15)$$

$$= \frac{1}{2} \times 75 \times 55$$

$$= 2062.5 \text{sq.m}$$

Area of the figure=Area of A+ Area of B+ Area of C+ Area of D

=3900+1250+312.5+2062.5

=7525sq.m

Solution 25:

Let x be the width of the footpath.

Then

$$\text{Area of footpath} = 2 \times (30 + 24) x + 4x^2$$

$$= 4x^2 + 108x$$

Again it is given that area of the footpath is 360sq.m.

Hence

$$4x^2 + 108x = 360$$

$$x^2 + 27x - 90 = 0$$

$$(x - 3)(x + 30) = 0$$

$$x = 3$$

Hence width of the footpath is 3m.

Solution 26:

Area of the square is 484.

Let a be the length of each side of the square.

Now

$$a^2 = 484$$

$$a = 22\text{m}$$

Hence length of the wire is=4x22=88m.

(i)

Now this 88m wire is bent in the form of an equilateral triangle.

$$\text{Side of the triangle} = \frac{88}{3}$$

$$= 29.3m$$

$$\text{Area of the triangle} = \frac{\sqrt{3}}{4} \times (\text{side})^2$$

$$= \frac{\sqrt{3}}{4} \times (29.3)^2$$

$$= 372.58\text{m}^2$$

(ii)

Let x be the breadth of the rectangle.

Now

$$2(l + b) = 88$$

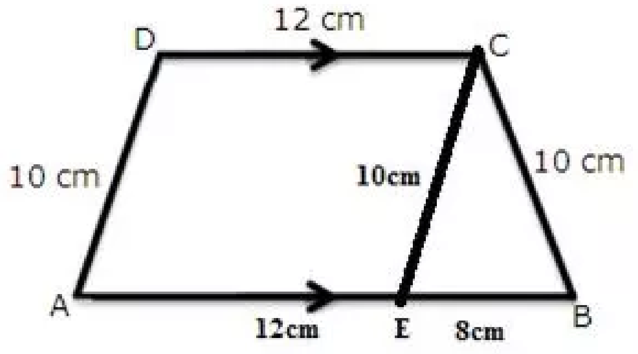
$$16 + x = 44$$

$$x = 28\text{m}$$

Hence area=16x28=448m2

Solution 27:

(i)



$$Area\ of\ \triangle EBC = \frac{1}{4} \times 8 \times \sqrt{4 \times 10^2 - 8^2}$$

$$= \frac{1}{4} \times 8 \times 18.3$$

$$= 36.6\ cm^2$$

Again

$$\text{Area of } \Delta EBC = \frac{1}{2} \times 8 \times h$$

$$36.6 = 4h$$

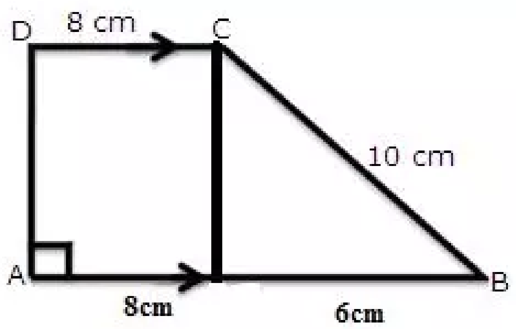
$$h = 9.15$$

$$\text{Area of ABCD} = \frac{1}{2} \times (12 + 20) \times 9.15$$

$$= \frac{1}{2} \times 32 \times 9.15$$

$$= 146.64 \text{sq.cm}$$

(ii)

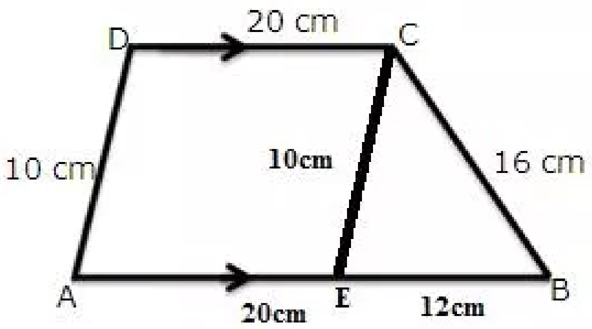


$$\text{Area of ABCD}=\frac{1}{2}\times(8+14)\times\left(\sqrt{10^2-6^2}\right)$$

$$=\frac{1}{2}\times 22\times 8$$

$$=88\text{sq.cm}$$

(iii)



For the triangle EBC,

S=19cm

$$\text{Area of } \Delta EBC=\sqrt{19\times(19-16)\times(19-12)\times(19-10)}$$

$$=\sqrt{19\times 3\times 7\times 9}$$

$$=59.9\text{ sq.cm}$$

Let h be the height.

$$Area\ of\ \triangle EBC = \frac{1}{2} \times 12 \times h$$

$$\Rightarrow 59.9 = 6h$$

$$\Rightarrow h = \frac{59.9}{6} = 9.98\ cm$$

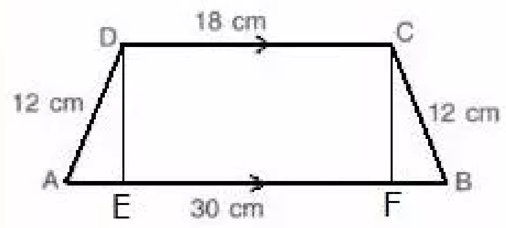
$$Area\ of\ ABCD = \frac{1}{2} \times (20 + 32) \times 9.98$$

$$= \frac{1}{2} \times 52 \times 9.98$$

$$= 259.48\ cm^2$$

In the given figure, we can observe that the non-parallel sides are equal and hence it is an isosceles trapezium.

Therefore, let us draw DE and CF perpendiculars to AB.



Thus, the area of the parallelogram is given by

Since AB = AE + EF + FB and CD = EF = 18 cm, we have

30 = AE + 18 + FB

$\Rightarrow$ 30 = AE + 18 + AE

$\Rightarrow$ 2AE + 18 = 30

$\Rightarrow$ 2AE = 30 − 18

$\Rightarrow$ 2AE = 12

$\Rightarrow$ AE = 6 cm

Now, consider the right angled triangle ADE.

$AD^2 = AE^2 + DE^2$

$\Rightarrow 12^2 = 6^2 + DE^2$

$\Rightarrow 144 = 36 + DE^2$

$\Rightarrow DE^2 = 144 - 36$

$\Rightarrow DE^2 = 108$

$\Rightarrow DE = \sqrt{36 \times 3}$

$\Rightarrow DE = 6\sqrt{3}$

Area (▱ABCD) = Area(△ADE) + Area(▱DEFC) + Area(△CFB)

$$\Rightarrow Area(▱ABCD) = \frac{1}{2} \times 6 \times 6\sqrt{3} + 18 \times 6\sqrt{3} + \frac{1}{2} \times 6 \times 6\sqrt{3}$$

$$\Rightarrow Area(▱ABCD) = 6 \times 6\sqrt{3} + 18 \times 6\sqrt{3}$$

$$\Rightarrow Area(▱ABCD) = 144\sqrt{3} = 249.41 cm^2$$

Solution 28:

Let b be the breadth of rectangle. then its perimeter

$$2(x+b)=70$$
$$x+b=35$$
$$b=35-x$$

Again

$$x\times b=300$$
$$x(35-x)=300$$
$$x^2-35x+300=0$$
$$(x-15)(x-20)=0$$
$$x=15,20$$

Hence its length is 20cm and width is 15cm.

Solution 29:

Let b be the width of the rectangle.

$$x\times b=640$$
$$b=\frac{640}{x}$$

Again perimeter of the rectangle is 104m.

Hence

$$2\left(x+\frac{640}{x}\right)=104$$
$$x^2-52x+640=0$$
$$(x-32)(x-20)=0$$
$$x=32,20$$

Hence

length=32m

width=20m.

Solution 30:

Let a be the length of the sides of the square.

According to the question

$$2a \times (a+6) = 3a^2$$

$$2a^2 + 12a = 3a^2$$

$$a = 12$$

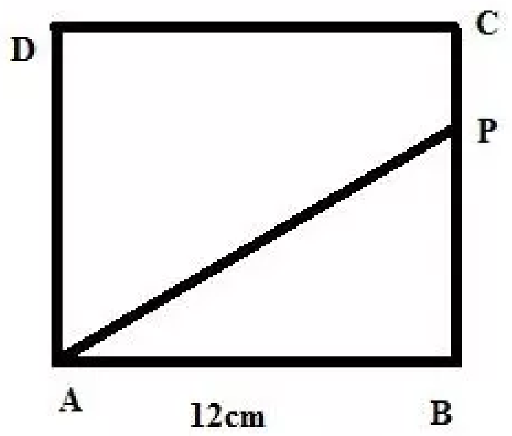
Hence sides of the square are 12cm each and

Length of the rectangle =2a=24cm

Width of the rectangle=a+6=18cm.

Solution 31:

The figure is shown below:



$$\frac{\text{Area of } \Delta ABP}{\text{Area of trapezium } APCD} = \frac{1}{5}$$

$$\Rightarrow \frac{\frac{1}{2} \times 12 \times (12 - CP)}{\frac{1}{2} \times (12 + CP) \times 12} = \frac{1}{5}$$

$$\Rightarrow 60 - 5CP = 12 + CP$$

$$\Rightarrow 6CP = 48$$

$$\Rightarrow CP = 8 \text{ cm}$$

Solution 32:

Length of the wall=45+2=47m

Breath of the wall=30+2=32m

Hence area of the inner surface of the wall is given by

$$A = (47 \times 2 \times 2.4) + (32 \times 2 \times 2.4)$$
$$= 225.6 + 153.6$$
$$= 379.2\ m^2$$

Solution 33:

Let a be the length of each side.

$$a^2 = 576$$

$$a = 24\text{cm}$$

$$4a = 96\text{cm}$$

Hence length of the wire=96cm

(i)

For the equilateral triangle,

$$side = \frac{96}{3} = 32\text{cm}$$

$$Area = \frac{\sqrt{3}}{4}(side)^2$$

$$= \frac{\sqrt{3}}{4} \times 32^2$$

$$= 256\sqrt{3}\text{sq.cm}$$

(ii)

Let the adjacent side of the rectangle be x and y cm.

Since the perimeter is 96 cm, we have,

$$2(x + y) = 96$$

Hence

$$x + y = 48$$

$$x - y = 4$$

$$x = 26$$

$$y = 22$$

Hence area of the rectangle is = 26 x 22 = 572 sq.cm

Solution 34:

Let 'y' and 'h' be the area and the height of the first parallelogram respectively.

Let 'height' be the height of the second parallelogram

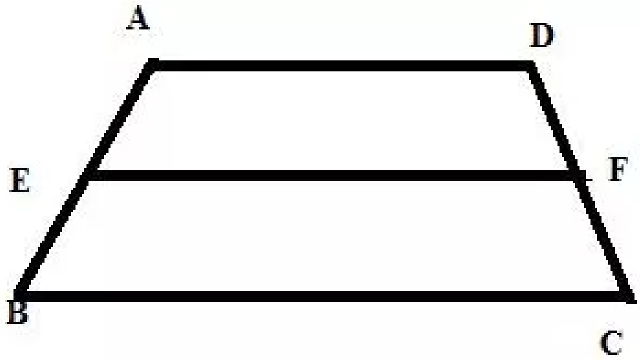
base of the first parallelogram= $\frac{y}{h}$ cm

base of second parallelogram= $\left(\frac{y}{h}+x\right)$ cm

$$\left(\frac{y}{h}+x\right)\times height = 2y$$

$$height = \frac{2hy}{y+hx}$$

Solution 35:



$$EF = \frac{1}{2}\times(AD+BC) = 26\,\text{cm}$$

Area of the trapezium= $\frac{1}{2}\times(AD+BC)\times h$

$$= 26\times 15$$

$$= 390\,\text{cm}^2$$

Solution 36:

Let a and b be the sides of the rectangle

Since the perimeter is 92 m, we have,

$2(a+b)=92$

$\Rightarrow a+b=46\ m....(1)$

Also given that diagonal of a trapezium is 34 m.

$\Rightarrow a^2+b^2=34^2....(2)$

We know that

$(a+b)^2-a^2-b^2=2ab$

From equations (1) and (2), we have,

$46^2-34^2=2ab$

$\Rightarrow 2ab=960$

$\Rightarrow ab=\frac{960}{2}$

$\Rightarrow ab=480\ m^2$

Exercise 20(C)

Solution 1:

Let r be the radius of the circle.

(i)

$2r = 28\text{cm}$

$\text{circumference} = 2\pi r$

$= 28\pi\text{cm}$

(ii)

$\text{area} = \pi r^2$

$= \pi\left(\frac{28}{2}\right)^2$

$= 196\pi\text{cm}^2$

Solution 2:

Let r be the radius of the circular field

(*i*)

$2\pi r = 308$

$\Rightarrow r = \dfrac{308}{2\pi}$

$\Rightarrow r = \dfrac{308}{2} \times \dfrac{7}{22}$

$\Rightarrow r = 49 \text{ m}$

(ii)

$\text{area} = \pi r^2$

$= \dfrac{22}{7} \times (49)^2$

$= 7546 \text{ m}^2$

Solution 3:

Let r be the radius of the circle.

$2\pi r + 2r = 116$

$2r(\pi + 1) = 116$

$r = \dfrac{116}{2(\pi + 1)}$

$= 14\text{cm}$

Solution 4:

Circumference of the first circle

$S_1 = 2\pi \times 25$

$= 50\pi\text{cm}$

Circumference of the second circle

$S_2 = 2\pi \times 18$

$= 36\pi\text{cm}$

Let r be the radius of the resulting circle.

$2\pi r = 50\pi + 36\pi$

$2\pi r = 86\pi$

$r = \dfrac{86\pi}{2\pi}$

$= 43\text{cm}$

Solution 5:

Circumference of the first circle

$$S_1 = 2\pi \times 48$$
$$= 96\pi \text{cm}$$

Circumference of the second circle

$$S_2 = 2\pi \times 13$$
$$= 26\pi \text{cm}$$

Let r be the radius of the resulting circle.

$$2\pi r = 96\pi - 26\pi$$
$$2\pi r = 70\pi$$
$$r = \frac{70\pi}{2\pi}$$
$$= 35\text{cm}$$

Hence area of the circle

$$A = \pi r^2$$
$$= \pi \times 35^2$$
$$= 3850\text{cm}^2$$

Solution 6:

Let the area of the resulting circle be r.

$$\pi \times (16)^2 + \pi \times (12)^2 = \pi \times r^2$$
$$256\pi + 144\pi = \pi \times r^2$$
$$400\pi = \pi \times r^2$$
$$r^2 = 400$$
$$r = 20\text{cm}$$

Hence the radius of the resulting circle is 20cm.

Solution 7:

Area of the circle having radius 85m is

$$A = \pi \times (85)^2$$
$$= 7225\pi m^2$$

Let r be the radius of the circle whose area is 49times of the given circle.

$$\pi r^2 = 49 \times (\pi \times 5^2)$$
$$r^2 = (7 \times 5)^2$$
$$r = 35$$

Hence circumference of the circle

$$S = 2\pi r$$
$$= 2\pi \times 35$$
$$= 220m$$

Solution 8:

Area of the rectangle is given by

$$A = 55 \times 42$$
$$= 2310 cm^2$$

For the largest circle, the radius of the circle will be half of the sorter side of the rectangle.

Hence r=21cm.

$$\text{Area of the circle} = \pi \times (21)^2$$
$$= 1384.74 cm^2$$
$$\text{Area remaining} = 2310 - 1384.74$$
$$= 925.26$$

Hence

$$\text{the volume of the circle: area remaining} = 1384.74 : 915.26$$
$$= 3:2$$

Solution 9:

Area of the square is given by

$$A = 28^2$$
$$= 784 \text{cm}^2$$

Since there are four identical circles inside the square.

Hence radius of each circle is one fourth of the side of the square.

$$\text{Area of one circle} = \pi \times 7^2$$
$$= 154 \text{cm}^2$$
$$\text{Area of four circle} = 4 \times 154 \text{cm}^2$$
$$= 616 \text{cm}^2$$
$$\text{Area remaining} = 784 - 616$$
$$= 168 \text{cm}^2$$

Area remaining in the cardboard is $= 168 \text{cm}^2$

Solution 10:

Let the radius of the two circles be 3r and 8r respectively.

$$\text{area of the circle having radius 3r} = \pi (3r)^2$$
$$= 9\pi r^2$$
$$\text{area of the circle having radius 8r} = \pi (8r)^2$$
$$= 64\pi r^2$$

According to the question

$$64\pi r^2 - 9\pi r^2 = 2695\pi$$
$$55r^2 = 2695$$
$$r^2 = 49$$
$$r = 7 \text{cm}$$

Hence radius of the smaller circle is $3 \times 7 = 21 \text{cm}$

Area of the smaller circle is given by

$$A = \pi r^2 = \frac{22}{7} \times 21^2 = 1386 \text{ cm}^2$$

Solution 11:

Let the diameter of the three circles be 3d, 5d and 6d respectively.

Now

$$\pi \times 3d + \pi \times 5d + \pi \times 6d = 308$$
$$14\pi d = 308$$
$$d = 7$$

$$\text{radius of the smallest circle} = \frac{21}{2} = 10.5$$
$$\text{Area} = \pi \times (10.5)^2$$
$$= 346$$
$$\text{radius of the largest circle} = \frac{42}{2} = 21$$
$$\text{Area} = \pi \times (21)^2$$
$$= 1385.5$$
$$\text{difference} = 1385.5 - 346$$
$$= 1039.5$$

Solution 12:

$$\text{Area of the ring} = \pi(20)^2 - \pi(15)^2$$
$$= 400\pi - 225\pi$$
$$= 175\pi$$
$$= 549.7\text{cm}^2$$

Solution 13:

Let r be the radius of the circular park.

$$2\pi r = 55$$
$$r = \frac{55}{2\pi}$$
$$= 8.75\text{m}$$

$$area\ of\ the\ park = \pi \times (8.75)^2 = 240.625\ \text{m}^2$$

Radius of the outer circular region including the path is given by

$$R = 8.75 + 3.5$$
$$= 12.25\ m$$

Area of that circular region is

$$A = \pi \times (12.25)^2 = 471.625\ \text{m}^2$$

Hence area of the path is given by

$$Area\ of\ the\ path = 471.625 - 240.625 = 231\ m^2$$

Solution 14:

Let r be the radius of the circular garden A.

Since the circumference of the garden A is 1.760 *Km* = 1760*m*, *we have*,

$$2\pi r = 1760\ \text{m}$$

$$\Rightarrow r = \frac{1760 \times 7}{2 \times 22} = 280\ \text{m}$$

$$\text{Area of garden A} = \pi r^2 = \frac{22}{7} \times 280^2\ \text{m}^2$$

Let R be the radius of the circular garden B.

Since the area of garden B is 25 times the area of garden A, we have,

$$\pi R^2 = 25 \times \pi r^2$$
$$\Rightarrow \pi R^2 = 25 \times \pi \times 280^2$$
$$\Rightarrow R^2 = 1960000$$
$$\Rightarrow R = 1400\ \text{m}$$

$$\text{Thus circumference of garden B} = 2\pi R = 2 \times \frac{22}{7} \times 1400 = 8800\ \text{m} = 8.8\ \text{Km}$$

Solution 15:

Diameter of the wheel = 84 cm

Thus, radius of the wheel = 42 cm

$$\text{Circumference of the wheel} = 2 \times \frac{22}{7} \times 42 = 264 \text{ cm}$$

In 264 cm, wheel is covering one revolution.

Thus, in 3.168 Km = 3.168 × 100000 cm, number of revolutions

$$\text{covered by the wheel} = \frac{3.168}{264} \times 100000 = 1200$$

Solution 16:

$$\text{the car travells in 10minutes} = \frac{66}{6}$$

$$= 11\text{km}$$

$$= 1100000\text{cm}$$

Circumference of the wheel = distance covered by the wheel in one revolution

Thus, we have,

$$\text{Circumference} = 2 \times \frac{22}{7} \times \frac{80}{2} = 251.43 \text{ cm}$$

Thus, the number of revolutions covered

$$\text{by the wheel in 1100000 cm} = \frac{1100000}{251.43} \approx 4375$$

Solution 17:

$$\text{radius of the wheel} = \frac{42}{2}$$

$$= 21\text{cm}$$

$$\text{circumference of the wheel} = 2\pi \times 21$$

$$= 132\text{cm}$$

$$\text{Distance travelled in one minute} = 132 \times 1200$$

$$= 158400\text{cm}$$

$$= 1.584\text{km}$$

$$\text{hence the speed of the train} = \frac{1.584\text{km}}{\frac{1}{60}\text{hr}}$$

$$= 95.04\text{km/hr}$$

Solution 18:

Time interval is $9.05 - 8.30 = 35\ minutes$

Area covered in one 60 minutes= $\pi \times 8^2 = 201 cm^2$

Hence area swept in 35 minutes is given by

$$A = \frac{201}{60} \times 35 = 117\frac{1}{3}\ cm^2$$

Solution 19:

Let R and r be the radius of the big and small circles respectively.

Given that the circumference of the bigger circle is 396 cm

Thus, we have,

$2\pi R = 396$ cm

$$\Rightarrow R = \frac{396 \times 7}{2 \times 22}$$

$\Rightarrow R = 63$ cm

Thus, area of the bigger circle $= \pi R^2$

$$= \frac{22}{7} \times 63^2$$

$$= 12474 \text{ cm}^2$$

Also given that the circumference of the smaller circle is 374 cm

$\Rightarrow 2\pi r = 374$

$$\Rightarrow r = \frac{374 \times 7}{2 \times 22}$$

$\Rightarrow r = 59.5$ cm

Thus, the area of the smaller circle $= \pi r^2$

$$= \frac{22}{7} \times 59.5^2$$

$$= 11126.5 \text{ cm}^2$$

Thus the area of the shaded portion $= 12474 - 11126.5 = 1347.5 \text{ cm}^2$

Solution 20:

From the given data, we can calculate the area of the outer circle and then the area of inner circle and hence the width of the shaded portion.

Given that the circumference of the outer circle is 132 cm

Thus, we have, $2\pi R = 132$ cm

$$\Rightarrow R = \frac{132 \times 7}{2 \times 22}$$

$$\Rightarrow R = 21 \text{ cm}$$

Area of the bigger circle $= \pi R^2$

$$= \frac{22}{7} \times 21^2$$

$$= 1386 \text{ cm}^2$$

Also given the area of the shaded portion.

Thus the area of the inner circle = Area of the outer circle − Area of the shaded portion

$$= 1386 - 770$$

$$= 616 \text{ cm}^2$$

$$\Rightarrow \pi r^2 = 616$$

$$\Rightarrow r^2 = \frac{616 \times 7}{22}$$

$$\Rightarrow r^2 = 196$$

$$\Rightarrow r = 14 \text{ cm}$$

Thus, the width of the shaded portion $= 21 - 14 = 7$ cm

Solution 21:

Let the radius of the field is *r* meter.

Therefore circumference of the field will be: $2\pi r$ meter.

Now the cost of fencing the circular field is 52,800 at rate 240 per meter.

Therefore

$$2\pi r \cdot 240 = 52800$$

$$r = \frac{52800 \times 7}{2 \times 240 \times 22}$$

$$= 35$$

Thus the radius of the field is 35 meter.

Now the area of the field will be:

$$\pi r^2 = \left(\frac{22}{7}\right) \cdot 35^2$$

$$= 3850 \text{ m}^2$$

Thus the cost of ploughing the field will be:

$$3850 \times 12.5 = 48,125 \text{ rupees}$$

Solution 22:

Let r and R be the radius of the two circles.

$$r + R = 10 \qquad \ldots(1)$$
$$\pi r^2 + \pi R^2 = 58\pi \qquad \ldots(2)$$

Putting the value of r in (2)

$$r^2 + R^2 = 58$$
$$(10 - R)^2 + R^2 = 58$$
$$100 - 20R + R^2 + R^2 = 58$$
$$2R^2 - 20R + 42 = 0$$
$$R^2 - 10R + 21 = 0$$
$$(R-3)(R-7) = 0$$
$$R = 3, 7$$

Hence the radius of the two circles is 3cm and 7cm respectively.

Solution 23:

From the figure:

$$AB = 28\text{cm}$$
$$BC = 21\text{cm}$$
$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{28^2 + 21^2}$$
$$= 35\text{cm}$$

Hence diameter of the circle is 35cm and hence

$$\text{Area} = \pi \times \left(\frac{35}{2}\right)^2$$
$$= 962.5 \ \text{cm}^2$$

$$\text{Area of the rectangle} = 28 \times 21$$
$$= 588\text{cm}^2$$

Hence area of the shaded portion is given by

$$A = 962 - 588 = 374.5 \ \text{cm}^2$$

Solution 24:

Since the diameter of the circle is the diagonal of the square inscribed in the circle.

Let a be the length of the sides of the square.

Hence

$$\sqrt{2}a = 2\times 7$$
$$a = \sqrt{2}\times 7$$
$$a^2 = 98$$

Hence the area of the square is 98sq.cm.

Solution 25:

Let a be the length of the sides of the equilateral triangle.

$$\frac{\sqrt{3}}{4}a^2 = 484\sqrt{3}$$
$$a^2 = 1936$$
$$a = 44\text{cm}$$
$$4a = 176\text{cm}$$

Hence the length of the wire is 176cm.

Let r be the radius of the circle.

Hence

$$2\pi r = 176$$
$$r = 28$$
$$\pi r^2 = 2464\ \text{cm}^2$$

Hence the area of the circle is $2464\ \text{cm}^2$

Solution 26:

Given the diameter of the front and rear wheels are
63 cm = 0.63 m and 1.54 m respectively.

Radius of the rear wheel $= \frac{1.54}{2} = 0.77$ m

and radius of the front wheel $= \frac{0.63}{2} = 0.315$ m

Distance travelled by tractor in one revolution of rear wheel
= circumference of the rear wheel
$= 2\pi r$
$= 2 \times \frac{22}{7} \times 0.77 = 4.84$ m

The rear wheel rotates at $24\frac{6}{11}$ revolutions per minute

$= \frac{270}{11}$ revolutions per minute

Since in one revolution the distance travelled by the rear wheel = 4.84 m

So, in $\frac{270}{11}$ revolutions, the tractor travels $\frac{270}{11} \times 4.84 = 118.8$ m

Let the number of revolutions made by the front wheel be x.

(i) Now, number of revolutions made by the front wheel in one minute
× circumference of the wheel
= the distance travalled by the tractor in one minute

$\Rightarrow x \times 2 \times \frac{22}{7} \times 0.315 = 118.8$

$\Rightarrow x = \frac{118.8 \times 7}{2 \times 22 \times 0.315} = 60$

(ii) Distance travelled by the tractor in 40 minutes
= Number of revolutions made by the rear wheel in 40 minutes
× circumference of the rear wheel

$= \frac{270}{11} \times 40 \times 4.84 = 4752$ m

Solution 27:

Let the radius of the circles be r_1 and r_2.

So, $r_1 + r_2 = 12 \Rightarrow r_2 = 12 - r_1$

Sum of the areas of the circles $= 74\pi$

$\Rightarrow \pi r_1^2 + \pi r_2^2 = 74\pi$

$\Rightarrow r_1^2 + r_2^2 = 74$

$\Rightarrow r_1^2 + (12 - r_1)^2 = 74$

$\Rightarrow r_1^2 + 144 - 24r_1 + r_1^2 = 74$

$\Rightarrow 2r_1^2 - 24r_1 + 70 = 0$

$\Rightarrow r_1^2 - 12r_1 + 35 = 0$

$\Rightarrow (r_1 - 7)(r_1 - 5) = 0$

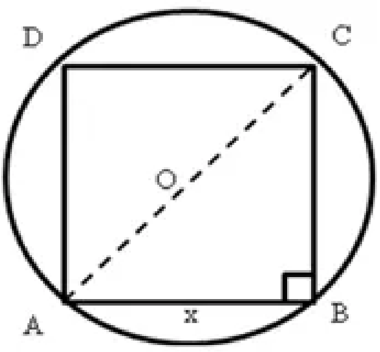
$\Rightarrow r_1 = 7$ or $r_1 = 5$

If $r_1 = 7$ cm, then $r_2 = 5$ cm

If $r_1 = 5$ cm, then $r_2 = 7$ cm

So, the diameters of the circles will be 10 cm and 14 cm.

Solution 28:



If AB = x, AC = $x\sqrt{2}$

Diameter of the circle = diagonal of the square

$\Rightarrow 2r = x\sqrt{2}$

$\Rightarrow r = \frac{x\sqrt{2}}{2}$

Area of the circle = πr^2

$$= \pi\left(\frac{x\sqrt{2}}{2}\right)^2$$

$$= \pi\left(\frac{x^2 2}{4}\right)$$

$$= \frac{\pi x^2}{2}$$

Area of the square = x^2

$$\text{Required ratio} = \frac{\frac{\pi x^2}{2}}{x^2}$$

$$= \frac{\pi}{2}$$

$$= \frac{22}{7} \times \frac{1}{2}$$

$$= \frac{11}{7}$$

Hence, the required ratio is 11 : 7.